PERIOD-III

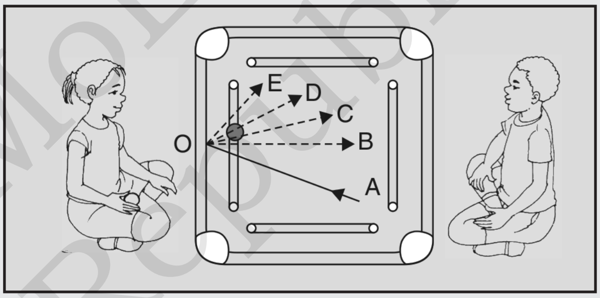
TOPIC
5

Plane Geometry

5.1 MEASURING *AND* DRAWING ANGLES

ACTIVITY 1

The following figure shows some of the angles a striker can make on a carrom board.



Name all the angles shown in this figure.

You should get 10 angles. Five of these are:

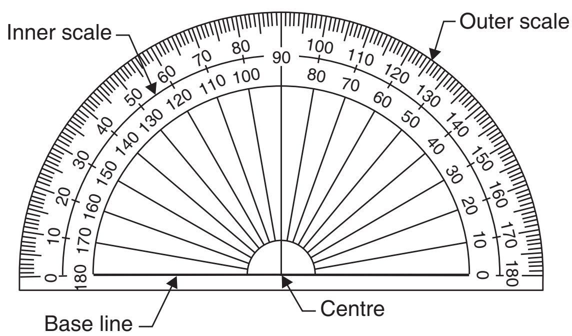
∠AOB, ∠AOC, ∠AOD, ∠AOE, ∠BOC.

Write the other five angles.

By this activity, we observe that angles are formed when two lines meet.

Introduction to the Various Parts of the Protractor

If you look at the protractor carefully, you will see that there are two sets of measurements written on it, *i.e.,* divisions marked in opposite directions. These are called *scales*. There is an *inner scale* and an *outer scale*, both having 0° to 180° in different directions.

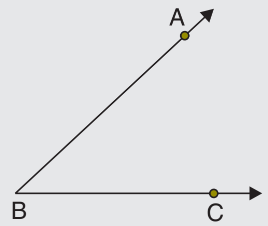


You can use a protractor to measure angles.

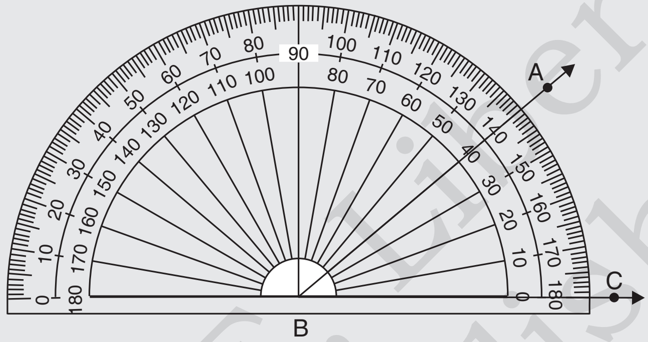
ACTIVITY 2

Measuring Angles Using the Protractor

Suppose you want to measure an angle ABC.



Given ∠ ABC



Measuring ∠ ABC

Step 1: Place the protractor so that the centre point of the baseline lies on the vertex B of the angle.

Step 2: Adjust the protractor (without shifting the centre from the vertex) so that one arm BC of the angle is along the baseline.

Step 3: There are two '*scales*' on the protractor: Read that scale which has the 0° mark coinciding with the baseline (inner scale in this example)

Step 4: The mark shown by BA on the curved edge gives the degree measure of the angle, *i.e.*, read the measure of this angle where the other arm BA crosses the scale. We write $m\angle ABC = 40°$, or simply $\angle ABC = 40°$.

Note: The length of the arms does not affect the measure of the angle.

ACTIVITY 3

Drawing Angles Using the Protractor

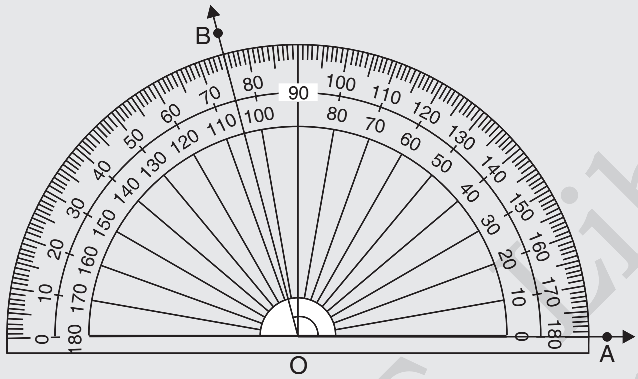
You can draw angles using the protractor.

Suppose you want to draw an angle of 105° using the protractor.

Step 1: Draw a ray OA.

Step 2: Place the protractor in such a way that its centre lies exactly at O and the base line lies along OA.

Step 3: Starting from 0° on the side of A, move the eyes and look for the 105° mark on the protractor. Mark a point B against this 105° mark.



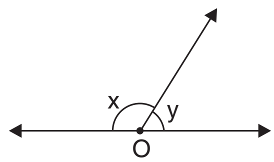
Step 4: Remove the protractor and draw the ray OB. Then, ∠AOB is the required angle whose measure is 105°.

Angles on a Straight Line

Draw a straight line to a point O on a line as shown in the following figure.

Now, measure the two angles formed using the protractor.

$$x = 122°, \quad y = 58°$$



Let us add their results,

$$x + y = 122° + 58° = 180°$$

Thus, $x + y = 180°$.

The sum of angles on a straight line is 180°, *i.e.,* the angles are supplementary. A pair of supplementary angles form a *linear pair* (180°) when placed adjacent to each other.

Hence, if the sum of two or more adjacent angles is 180°, then the non-common arms of the angles form a straight angle.

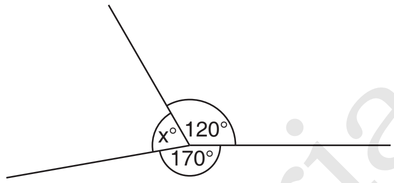
Note: Angles measuring 180° are called straight angles.

5.2 CALCULATING ANGLES

Angles at a Point and on a Line

One complete revolution is equivalent to a rotation of 360° about a point. Similarly, half a complete revolution is equivalent to a rotation of 180° about a point. These facts can be seen clearly by looking at *either* a circular angle measurer *or* a semi-circular protractor.

Example 1. *Calculate the size of the angle x in the adjacent figure.*



Solution. The sum of all the angles around a point is 360°.

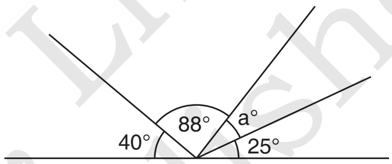
$\therefore \quad 120° + 170° + x = 360°$

$\Rightarrow \quad x = 360° - 120° - 170°$

$x = 70°$

Therefore angle x is 70°.

Example 2. *Calculate the size of angle a in the adjacent diagram.*



Solution. The sum of all the angles at a point on a straight line is 180°.

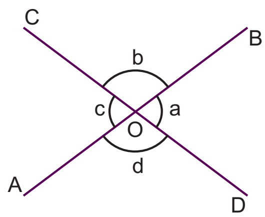
$\therefore \quad 40° + 88° + a + 25° = 180°$

$\Rightarrow \quad a = 180° - 40° - 88° - 25°$

$a = 27°$

Therefore angle a is 27°.

Vertically Opposite Angles



When two straight lines intersect each other, they form four angles. A pair of angles with no common arm are called vertically opposite angles (*abbreviated as vert. opp. ∠s*).

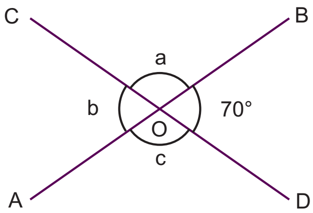
In the figure, a and c are vertically opposite angles and so are angles b and d.

If two straight lines intersect, then the vertically opposite angles are equal.

Thus, $\angle a = \angle c$ and $\angle b = \angle d$.

Example 3. *In the figure, two straight lines AB and CD intersect at O. Find the angles a, b and c.*

Solution. Angles COB and DOB form a linear pair.



Therefore,

$\angle COB + \angle DOB = 180°$

$\Rightarrow \quad a + 70° = 180°$

$\Rightarrow \quad a = 180° - 70° = 110°$

As $\angle AOC = \angle BOD$ (vert. opp. $\angle s$)

$\Rightarrow$ $b = 70°$

$\angle AOD = \angle COB$ (vert. opp. $\angle s$)

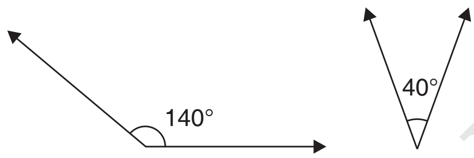
$\Rightarrow$ $c = a = 110°$

Hence, $a = 110°$, $b = 70°$ and $c = 110°$.

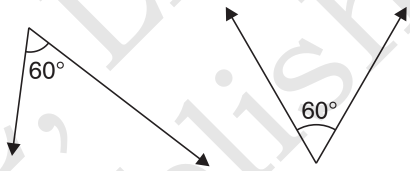
EXERCISE 5.1

1. Check which of the following pairs of angles form a linear pair:

(*a*)

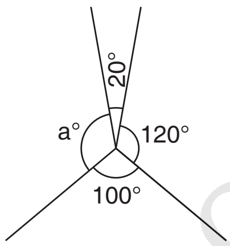


(*b*)

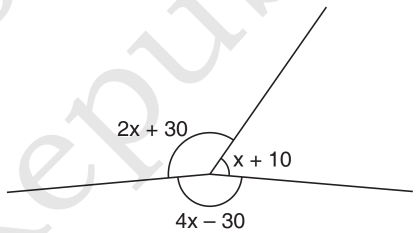


2. Find the value of the lettered angles in the following diagrams.

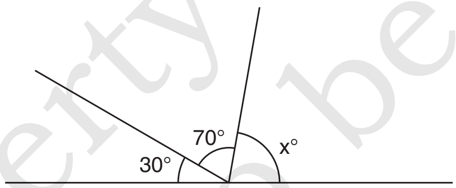
(*a*)



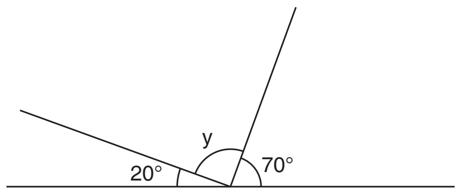
(*b*)



3. Find the value of x in the figure below.



Q. 3



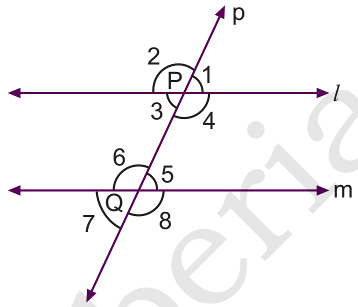
Q. 4

4. Find the value of y in the above figure.

5.3 ANGLE PROPERTIES OF PARALLEL LINES

A line that intersects two or more lines (not necessarily parallel lines) at *distinct* points is called a *transversal.* A transversal to two parallel lines is of special significance in the study of geometry.

If two parallel lines l and m are cut by a transversal p at points P and Q respectively, then eight angles marked 1 to 8 are formed as shown in the adjacent figure. These angles have special names, individually as well as in pairs. These pairs of angles have special properties. If any one of these eight angles is known, then the remaining seven can be easily obtained.



Now we state some important properties of these angles. If two parallel lines are cut by a transversal, then

(*i*) pairs of corresponding angles are equal,

i.e., $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$ and $\angle 4 = \angle 8$

(*ii*) pairs of alternate interior angles are equal,

i.e., $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$

(*iii*) pairs of alternate exterior angles are equal

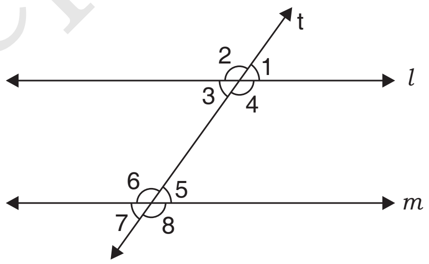
i.e., $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$

(*iv*) the sum of interior angles on the same side of the transversal is 180°,

i.e., $\angle 3 + \angle 6 = 180°$

and $\angle 4 + \angle 5 = 180°$

Example 4. *In the following diagram line $l \parallel m$ and t is a transversal. If $\angle 1 = 30°$, find all the angles from 2 to 8.*



Solution. Given $\angle 1 = 30°$.

(*i*) $\angle 1$ and $\angle 5$ are corresponding angles.

$\therefore$ $\angle 5 = \angle 1 \Rightarrow \angle 5 = 30°$ [$\because \angle 1 = 30°$]

(*ii*) $\angle 1 + \angle 2 = 180°$ [Linear pair]

$\Rightarrow$ $\angle 2 = 180° - \angle 1 \Rightarrow 180° - 30° = 150°$

[$\because \angle 1 = 30°$]

(*iii*) $\angle 2 = \angle 4$ [Vertically opposite angles]

$\Rightarrow$ $\angle 4 = 150°$ [$\because \angle 2 = 150°$]

(*iv*) $\angle 1 = \angle 3$ [Vertically opposite angles]

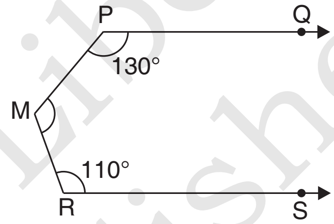
$\Rightarrow$ $\angle 3 = 30°$ [$\because \angle 1 = 30°$]

(*v*) $\angle 4 = \angle 6$ [Alternate interior angles]

$\Rightarrow$ $\angle 6 = 150°$ [$\because \angle 4 = 150°$]

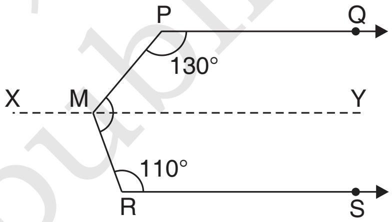
(*vi*)	$\angle 3 = \angle 7$	[Corresponding angles]
$\Rightarrow$	$\angle 7 = 30°$	[$\because$ $\angle 3 = 30°$]
(*vii*)	$\angle 4 = \angle 8$	[Corresponding angles]
$\Rightarrow$	$\angle 8 = 150°$	[$\because$ $\angle 4 = 150°$]
Thus,	$\angle 1 = \angle 3 = \angle 5 = \angle 7 = 30°$	
	$\angle 2 = \angle 4 = \angle 6 = \angle 8 = 150°$.	

Example 5. *In the given diagram PQ || RS, ∠QPM = 130° and ∠SRM = 110°. Find ∠PMR.*



Solution.

Construction. Through M, draw a line XY || PQ, and PM is a transversal.



$\therefore$ $\angle QPM + \angle PMY = 180°$

[Sum of interior angles on the same side of a transversal is 180°]

$\Rightarrow$ $\angle PMY = 180° - 130°$ [$\because$ $\angle QPM = 130°$]

$= 50°$

Now, XY || PQ and PQ || RS $\Rightarrow$ XY || RS

XY || RS and MR is a transversal.

$\because$ $\angle SRM + \angle RMY = 180°$

[Sum of interior angles on the same side of a transversal is 180°]

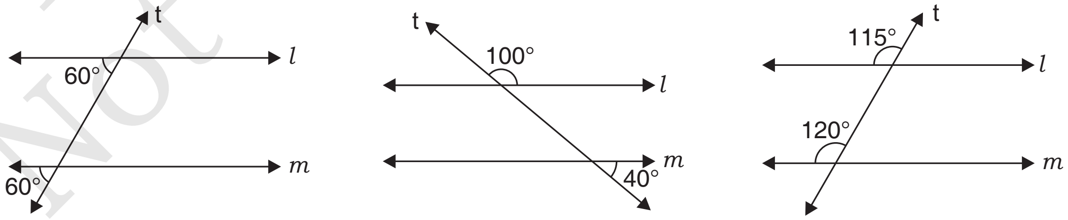
$\Rightarrow$ $\angle RMY = 180° - 110° = 70°$ [$\because$ $\angle SRM = 110°$]

$\Rightarrow$ $\angle PMR = \angle PMY + \angle RMY = 50° + 70° = 120°$

Hence, $\angle PMR = 120°$.

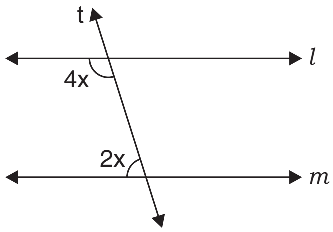
EXERCISE 5.2

1. In the figures given below, find out whether line *m* is parallel to line *l*.

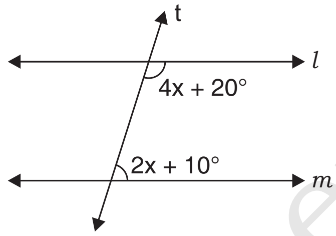


(a) (b) (c)

2. If line *l* is parallel to line *m*, find the value of *x*.

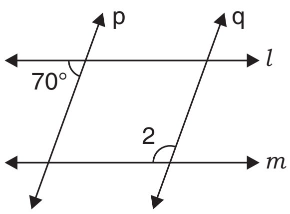


(a)

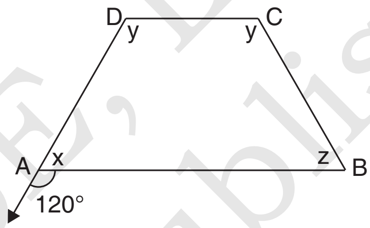


(b)

3. (*a*) In the diagram given below $l \parallel m$, $p \parallel q$ and $\angle 1 = 70°$. Find $\angle 2$.
(*b*) In the diagram given below AB || CD. Find the angles *x*, *y*, *z*.



(a)



(b)

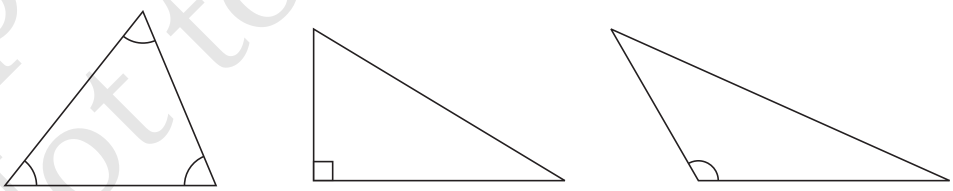
5.4 TRIANGLES

Name of Triangles

Triangles can be described in terms of their sides or their angles, or both.

(a) Naming Triangles Based on Angles

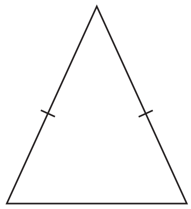
- An *acute-angled* triangle has *all its angles less than 90°.*
- A *right-angled* triangle has an angle of 90°.
- An *obtuse-angled* triangle has one angle greater than 90°.



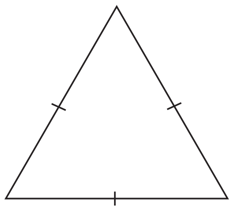
Acute-angled   Right-angled   Obtuse-angled

(b) Naming Triangles Based on Sides

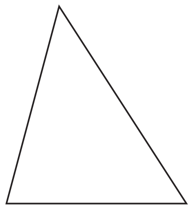
- An *isosceles* triangle has two sides of equal length, and the angles opposite the equal sides are equal.



Isosceles triangle



Equilateral triangle



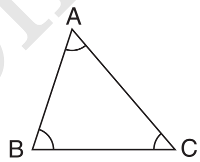
Scalene triangle

- An *equilateral* triangle has three sides of equal length and three equal angles.
- A *scalene* triangle has three sides of different lengths and all three angles are different.

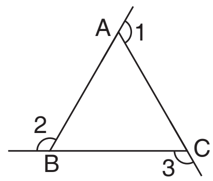
5.5 ANGLE PROPERTIES OF TRIANGLES

1. The sum of three interior angles of a triangle is always 180°. This property is called the *angle sum property* of a triangle. Therefore,

$$\angle A + \angle B + \angle C = 180°.$$

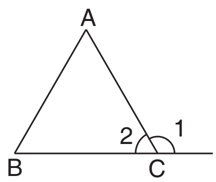


2. The exterior angles of a triangle always add up to 360°. Therefore $\angle 1 + \angle 2 + \angle 3 = 180°$.



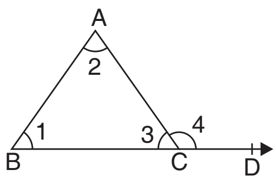
3. The sum of consecutive interior and exterior angle is supplementary. Therefore,

$$\angle 1 + \angle 2 = 180°.$$



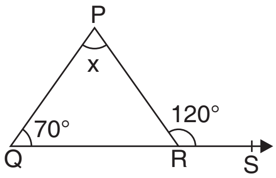
4. If one side of a triangle is extended, then the exterior angle so formed is equal to the sum of the two interior opposite angles. This property is called an *exterior angle property* of a triangle.

Therefore, $\angle 4 = \angle 1 + \angle 2.$

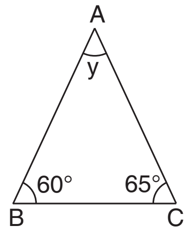


Example 6. *In the following diagrams, find the unknown angles:*

(*i*)



(*ii*)



Solution.

(*i*) In ΔPQR,

$\angle P + \angle Q = \angle PRS$ [Exterior angle property of a triangle]

∴ $x + 70° = 120° \Rightarrow x = 120° - 70° = 50°$

Thus, $\angle QPR = 50°$

(*ii*) In ΔABC,

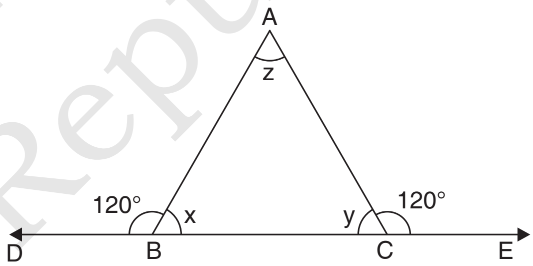
$\angle A + \angle B + \angle C = 180°$ [Sum of the angles of a triangle is 180°]

$\Rightarrow$ $y + 60° + 65° = 180°$

$\Rightarrow$ $y = 180° - 125° = 55°$

Thus, $\angle BAC = 55°$

Example 7. *In the given ΔABC, exterior angles ∠ACE = 120° and ∠ABD = 120°. Find y and z and identify the type of triangle in the given diagram.*



Solution. $120° + x = 180°$ [Linear pair at point B]

$\Rightarrow$ $x = 180° - 120° = 60°$ ...(1)

$120° + y = 180°$ [Linear pair at point C]

$\Rightarrow$ $y = 180° - 120° = 60°$ ...(2)

In ΔABC,

$x + y + z = 180°$ [Angle sum property of a Δ]

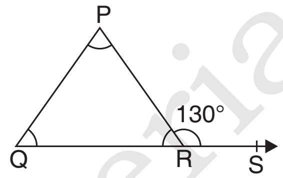
$\Rightarrow$ $60° + 60° + z = 180°$

[Substituting the values from (1) and (2)]

$\Rightarrow$ $z = 180° - 120° = 160°$

Since, all the angles of the triangle are 60° each, it is an equivalent triangle.

Example 8. *In ΔPQR, ∠P and ∠Q are in the ratio 2 : 3. Find all the angles of the triangle.*



Solution. Let the angles ∠P and ∠Q be $2x$ and $3x$ respectively.

∠PRQ + ∠PRS = 180°

[Linear pair at the point R]

⇒ ∠PRQ + 130° = 180°

⇒ ∠PRQ = 180° – 130° = 50°

In ΔPQR,

∠P + ∠Q + ∠R = 180° [Angle sum property of a Δ]

⇒ $2x + 3x + 50° = 180°$

⇒ $5x = 180° - 50° = 130°$ ⇒ $x = \dfrac{130°}{5} = 26°$

∴ $2x = 52°$ and $3x = 78$

Thus, ∠P = 52°, ∠Q = 78° and ∠R = 50°.

EXERCISE 5.3

1. Name the types of following triangles:

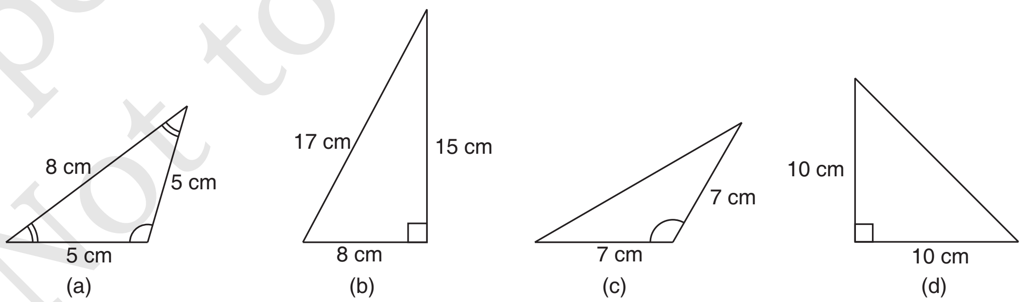
(*a*) Triangle with lengths of sides 7 cm, 8 cm and 9 cm.

(*b*) ΔABC with AB = 8.7 cm, AC = 7 cm and BC = 6 cm.

(*c*) ΔPQR such that PQ = QR = PR = 5 cm.

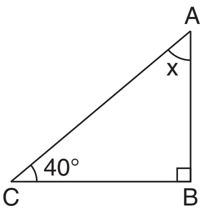
(*d*) ΔDEF with ∠D = 90°

2. Name each of the following triangles in two different ways: (you may judge the nature of the angle by observation)

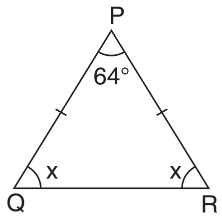


3. Find the value of *x*, *y*, *z* in the following triangles:

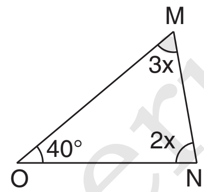
(*a*)



(*b*)

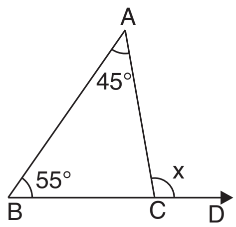


(*c*)

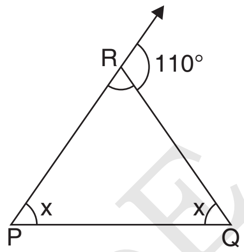


4. Find the value of *x* in the following triangles:

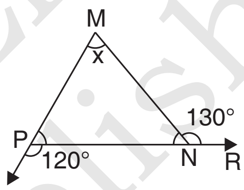
(*a*)



(*b*)



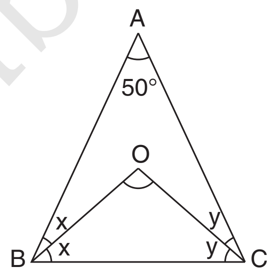
(*c*)



5. In the given diagram, find ∠BOC.

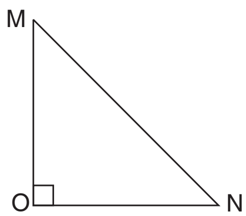
6. An exterior angle of a triangle is 120°. If interior opposite angles are in the ratio of 3 : 5, find the angles of the triangle.

7. If the angles of a triangle are in the ratio 2 : 3 : 4, find the angles.



Q. 5

5.6 RIGHT ANGLED TRIANGLES



- One angle is always 90° or right angle.
- The side opposite angle of 90° is the hypotenuse.
- The hypotenuse is always the longest side.
- The sum of the other two interior angles is equal to 90°.
- The other two sides adjacent to the right angle are called base and perpendicular.

5.7 SQUARES *AND* SQUARE ROOTS

Squares

If a number is multiplied by itself, the product obtained is called the *square* of that number.

For a given number 'x', the square of x is $(x \times x) = x^2$ we read it as x square.

For example: $5^2 = (5 \times 5) = 25$

We say that the square of 5 is 25.

Example 9. *Find the square of 36.*

Solution. Square of 36 = 36^2

$= 36 \times 36 = 1296$

$$\begin{array}{r} 36 \\ \times\ 36 \\ \hline 216 \\ 1080 \\ \hline 1296 \end{array}$$

Perfect Square Number

The numbers 1, 4, 9, 16, etc. can respectively be expressed as (1^2), (2^2), (3^2), (4^2), etc.

Thus 1, 4, 9, 16, etc. are known as square numbers or perfect squares.

In general, a natural number is called a square number or *perfect square* if it is the square of a natural number.

Example 10. *Is 1764 a perfect square? If so, find the number whose square is 1764.*

Solution. Resolving 1764 into prime factors, we get

$1764 = (2 \times 2 \times 3 \times 3 \times 7 \times 7)$

$= (2 \times 2) \times (3 \times 3) \times (7 \times 7)$

[Grouping the factors in pairs of identical factors]

$= 2^2 \times 3^2 \times 7^2$

$= (2 \times 3 \times 7)^2 = (42)^2$

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Hence, 1764 is a perfect square as it can be expressed as the product of pairs of equal factors.

So, 42 is the number whose square is 1764.

Square Roots

We know that 16, 64, 81 are perfect suqares.

$$16 = 4^2 = 4 \times 4$$
$$64 = 8^2 = 8 \times 8$$
$$81 = 9^2 = 9 \times 9$$

The square roots of 16, 64, 81 are 4, 8, 9 respectively.

Thus, square root of a number x is that number, which when multiplied by itself, gives x as the product.

The square root is a number x is denoted by $\sqrt[2]{x}$ or only $\sqrt{x}$.

$\sqrt{}$ symbol is known as radical sign.

Thus, $\sqrt{16} = 4$, $\sqrt{64} = 8$, $\sqrt{81} = 9$.

Square Root of a Perfect Square by Prime-Factorisation Method

To find the square root of a perfect square number we follow the given steps:

Step 1: Resolve the given number into prime factors.

Step 2: Make pairs of identical prime factors.

Step 3: Take one prime factor from every pair and find the product of these factors.

The product gives us the square root of the given number.

Example 11. *Find the square root of (i) 676 and (ii) 3025 by the method of prime factorisation.*

2	676
2	338
13	169
13	13
	1

Solution. (*i*) By prime factorisation, we get

$$676 = (2 \times 2) \times (13 \times 13)$$

$$\therefore \quad \sqrt{676} = 2 \times 13 = 26.$$

(*ii*) Resolving 3025 into prime factors, we get

5	3025
5	605
11	121
11	11
	1

$$3025 = (5 \times 5) \times (11 \times 11)$$

$$\therefore \quad \sqrt{3025} = 5 \times 11 = 55.$$

EXERCISE 5.4

1. Is 343 a perfect square?
2. Show that each of the following is a perfect square. Also find the number whose perfect square is the given number.

   (*a*) 625 (*b*) 7056 (*c*) 1521

3. By what least number should the given number be multiplied to get a perfect square number? In each case, find the number whose square is the new number.

(*a*) 252 (*b*) 4851 (*c*) 1452

4. By prime factorisation method, find the square root of these numbers.

(*a*) 36 (*b*) 64 (*c*) 900 (*d*) 576

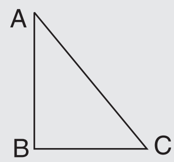
5.8 PYTHAGORAS THEOREM

Pythagoras was a Greek philosopher who lived from 580 BC to 500 BC.

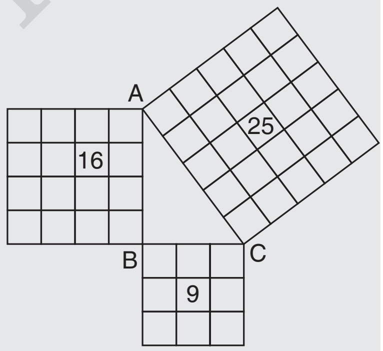
He gave a wonderful relation between the lengths of the sides of a right angle triangle. This relationship is now known as Pythagoras theorem.

ACTIVITY 4

1. Take a white chart paper and draw a right triangle with sides AB = 4 cm, BC = 3 cm and AC = 5 cm as shown in the figure.



2. Now, using a pair of scissors, cut three squares of dimensions 3 cm, 4 cm and 5 cm from a grid sheet and paste them along the sides of the triangle as shown in the figure.



3. Count the number of squares on each side.

Number of squares on side AB = $16 = 4^2$

Number of squares on side BC = $9 = 3^2$

Number of squares on side AC = $25 = 5^2$

Clearly, $16 + 9 = 25$

$$AB^2 + BC^2 = AC^2$$

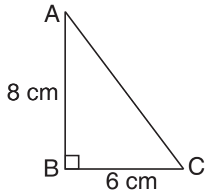
According to Pythagoras theorem in a right angle triangle, the square of the hypotenuse equals to the sum of the squares of its two remaining sides.

Converse of Pythagoras Theorem

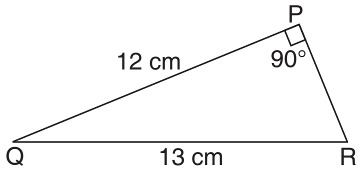
In a triangle, if sum of the squares of two sides is equal to the square of the hypotenuse, then the triangle is a right-angled triangle.

Example 13. *Find the third side of the following right-angled triangles.*

(*i*)



(*ii*)



Solution.

(*i*) In ΔABC, $\angle B$ is a right angle.

$\therefore \quad (AB)^2 + (BC)^2 = (AC)^2$ [According to Pythagoras theorem]

$\Rightarrow \quad (8)^2 + (6)^2 = (AC)^2 \quad \Rightarrow \quad 64 + 36 = (AC)^2$

$\Rightarrow \quad 100 = AC^2 \quad \Rightarrow \quad AC = \sqrt{100} = 10$

Side $\quad AC = 10$ cm.

(*ii*) In ΔPQR, $\angle P$ is a right angle.

$\therefore \quad (PQ)^2 + (PR)^2 = (QR)^2$ [According to Pythagoras theorem]

$\Rightarrow \quad (12)^2 + (PR)^2 = (13)^2 \quad \Rightarrow \quad 144 + (PR)^2 = 169$

$\Rightarrow \quad (PR)^2 = 169 - 144 \quad \Rightarrow \quad (PR)^2 = 25$

$\Rightarrow \quad PR = \sqrt{25} = 5$

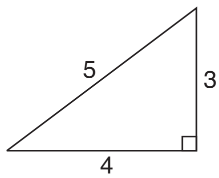
Side $\quad PR = 5$ cm.

5.9 PYTHAGOREAN TRIPLES

The simplest right angled triangle with sides of *integer* length is the *3-4-5 triangle.*

The numbers 3, 4 and 5 satisfy the rule $3^2 + 4^2 = 5^2$

The set of positive integers $\{a, b, c\}$ is a *Pythagorean triple* if it obeys the rule $a^2 + b^2 = c^2$.



Other examples are:

$\{5, 12, 13\}, \{7, 24, 25\}, \{8, 15, 17\}$.

Example 1. *Show that* $\{5, 12, 13\}$ *is a Pythagorean triple.*

Solution. We find the square of the *largest* number first

$13^2 = 169$

and $\quad 5^2 + 12^2 = 25 + 144 = 169$

$\therefore \quad 5^2 + 12^2 = 13^2$

So, $\{5, 12, 13\}$ is a Pythagorean triple.

Example 2. *Find k if {9, k, 15} is a Pythagorean triple.*

Solution. By Pythagoras theorem,

$$9^2 + k^2 = 15^2 \quad \text{[By Pythagoras theorem]}$$

$$\Rightarrow \quad 81 + k^2 = 225$$

$$\Rightarrow \quad k^2 = 144 \Rightarrow k = \sqrt{144} \quad \{\text{as } k > 0\}$$

$$\Rightarrow \quad k = 12.$$

EXERCISE 5.5

1. In the following right angle triangles, height (a) and base (b) are given. Find the hypotenuse (c).

(*a*) a = 6 cm b = 8 cm (*b*) a = 1.5 cm b = 2 cm

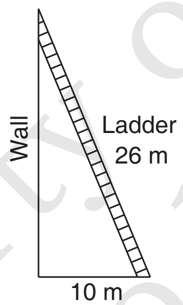
(*c*) a = 40 cm b = 9 cm (*d*) a = 8 cm b = 15 cm

2. Sides of the following triangles are given. Find out whether they are right angled triangles:

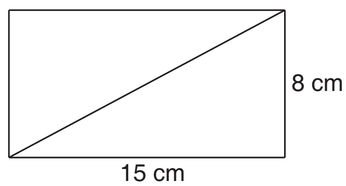
(*a*) 4 cm, 5 cm, 6 cm (*b*) 15 cm, 20 cm, 25 cm

(*c*) 6 cm, 8 cm, 11 cm (*d*) 5 cm, 12 cm, 13 cm

3. A ladder that is 26 m long, rests against a vertical wall, with its foot 10 m away from the wall. How high up the wall will the ladder reach?



Q. 3



Q. 4

4. The sides of a rectangle are 15 cm and 8 cm. Find the length of diagonal.

5. Which of the following are Pythagorean triples?

(*a*) {8, 15, 17} (*b*) {6, 8, 10} (*c*) {5, 6, 7} (*d*) {14, 48, 50}

6. Find k if the following are Pythagorean triples:

(*a*) {8, 15, k} (*b*) {k, 24, 26} (*c*) {14, k, 50}

5.10 PARALLELOGRAMS *AND* TRAPEZIUM–KITES, RHOMBUSES

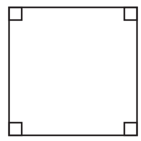
Since quadrilaterals have special names. Each special name has a special *property* related to it.

Note: You can often use these properties to find *lengths* and *angles* in quadrilaterals. Quadrilaterals can be sorted out into two main sets: those which are parallelograms and those which are not.

Parallelograms	Non-parallelograms
Parallelograms, rhombus, square and rectangle	Trapezium, kites and irregular quadrilaterals

Square

A square has:



- all angles equal to 90°
- all sides equal
- opposite sides parallel
- diagonals equal in length and bisect each other
- diagonals cross at right angles
- diagonals bisect corner angles

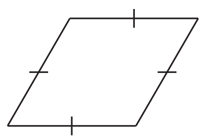
Rectangle

A rectangle has:

- all angles equal to 90°
- opposite sides equal
- opposite sides parallel
- diagonals equal in length and bisect each other

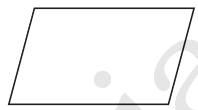
Rhombus

A rhombus has:



- all sides equal
- opposite sides parallel
- opposite angles equal
- diagonals bisect each other
- diagonals cross at right angles
- diagonals bisect corner angles

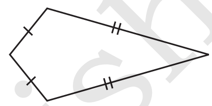
Parallelogram



A parallelogram has:

- opposite angles equal
- opposite sides equal
- opposite sides parallel
- diagonals bisect each other

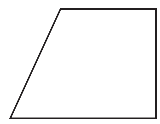
Kite



A kite has:

- one pair of opposite angles equal
- two pairs of adjacent sides equal
- diagonals cross at right angles
- only one diagonal is bisected
- only one pair of opposite angles is bisected

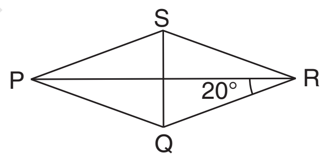
Trapezium



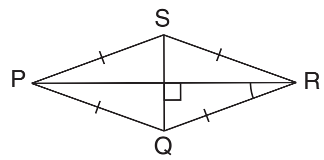
A trapezium has:

- one pair of opposite sides parallel

Example 14. *In the given figure, PQRS is a rhombus and ∠PRQ = 20°. What is the value of ∠QSR?*



Solution. A rhombus has all sides equal and the diagonals bisect each other at right angles.



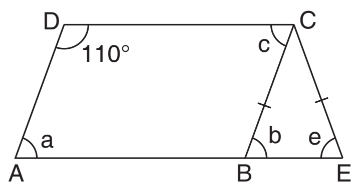
Therefore ∠RQS = 90° – 20° = 70°.

Triangle ∠QRS is isosceles,

Therefore ∠QSR = ∠RQS = 70°

(Base angles of an isosceles triangle are equal)

Example 15. *Find the size of each lettered angle in the given diagram.*



Solution. ABCD is a prallelogram and BCE is an isosceles triangle.

Opposite angles of a parallelogram are equal.

i.e., ∠ABC = 110°

Adjacent angles on a straight line add up to 180°.

i.e., $b + \angle ABC = 180°$

$\Rightarrow$ $b = 180° - 110° = 70°$

Base angles of an isosceles triangle are equal.

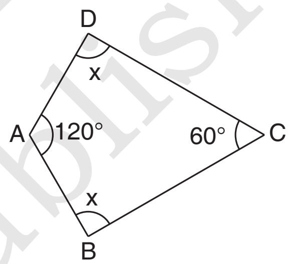
i.e., $e = b = 70°$

Alternate angles are equal. *i.e.*, $c = b = 70°$

Opposite angles of a parallelogram are equal,

i.e., $a = c = 70°$

Example 16. *Find the value of x in the given figure.*



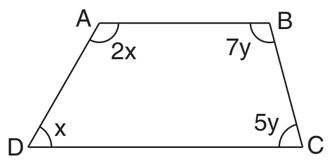
Solution. As we know that the sum of all angles of a quadrilateral (kite) is 360°. Therefore,

$120° + x + 60° + x = 360°$

$\Rightarrow$ $2x + 180° = 360°$

$\Rightarrow$ $2x = 360° - 180° = 180°$

$\Rightarrow$ $x = \dfrac{180°}{2} = 90°.$



Example 17. *ABCD is a trapezium such that AB || CD as shown in the adjacent figure:*

Find the angles of the trapezium.

Solution. In trapezium, the angles on the same side between the pair of parallel sides are supplementary *i.e.*, they are add up to 180°.

$\therefore$ $\angle A + \angle D = 180°$

$\Rightarrow$ $2x + x = 180°$

$\Rightarrow$ $3x = 180°$ $\Rightarrow$ $x = \dfrac{180°}{3} = 60°$

Similarly, $\angle B + \angle C = 180°$ $\Rightarrow$ $7y + 5y = 180°$

$\Rightarrow$ $12y = 180°$ $\Rightarrow$ $y = \dfrac{180°}{12} = 15°$

$\therefore$ Angles of the trapezium are:

$\angle A = 2x = 2 \times 60° = 120°$; $\angle B = 7y = 7 \times 15° = 105°$

$\angle C = 5y = 5 \times 15° = 75°$; $\angle D = x = 60°.$

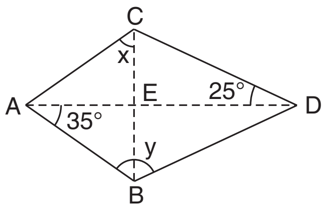
EXERCISE 5.6

1. In a parallelogram PQRS, if ∠QRS = $2x$, ∠PQR = $4x$ and ∠PSQ = $4x$, find the angles of the parallelogram.

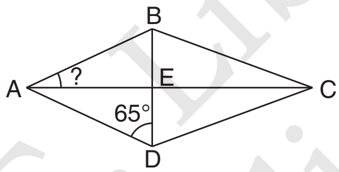
2. ABCD is a rhombus with ∠ABC = 50°. Determine ∠ACD.

3. Two consecutive angles of a prallelogram are $(x + 60)°$ and $(2x + 30)°$. What special name can you give to this parallelogram?

4. Find the values of x and y in the given figure.



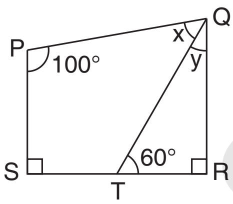
Q. 4



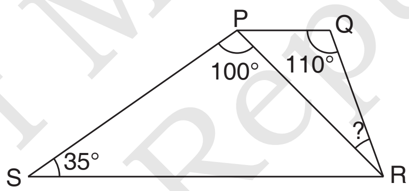
Q. 5

5. If ∠ADE = 65°, what is ∠BAE?

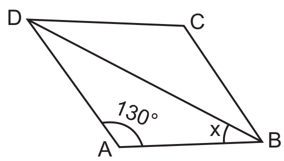
6. Given figure is a trapezium in which PS || QR. Find the values of x and y.



Q. 6



Q. 7



Q. 8

7. PQRS is a trapezium in which PQ || SR. Find the unknown angle ∠QRP.

8. In the above figure, ABCD is a rhombus. Find the value of x.

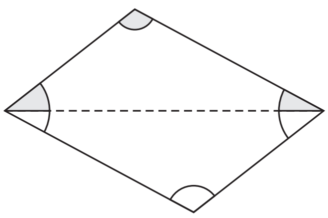
5.11 POLYGONS

Any closed figure/curve made up of only line segments is called a polygon. Triangle, quadrilateral, pentagon etc. are examples of polygons.

The Interior Angles of a Polygon

Now consider finding the sum of angles of a quadrilateral.

We can construct a diagonal of the quadrilateral as shown.



The constructed diagonal divide the quadrilateral into two triangles.

We have already seen that the sum of interior angles of a triangle is 180°.

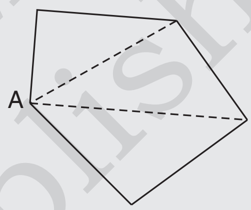
So, the sum of the interior angles of a quadrilateral

= 2 × Sum of interior angles of a triangle

$= 2 \times 180° = 360°$

We can generalise this process to find the sum of the interior angles of any polygon.

ACTIVITY 5



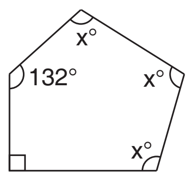
1. Draw any pantagon (5-sided polygon) and label one of its vertices A. Draw in all the diagonals from A.
2. Repeat 1 for a hexagon, a heptagon (7-gon), an octagon, and so on, drawing diagonals from one vertex only.
3. Copy and complete the following table:

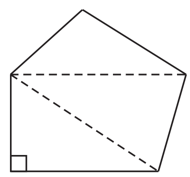
Polygon	Number of sides	Number of diagonals from A	Number of triangles	Angle sum of polygon
Quadrilateral	4	1	2	$2 \times 180° = 360°$
Pentagon				
Hexagon				
Octagon				
20-gon				

We have discovered that:

The sum of the interior angles of any n-sided polygon is $(n - 2) \times 180°$.

Example 18. *Find x, giving a brief reason.*





Solution. The pentagon has 5 sides.

$\therefore$ Sum of interior angles = $(5 - 2 = 3) \times 180° = 540°$

i.e., $x + x + x + 132° + 90° = 540°$

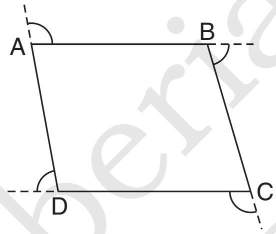
$\Rightarrow 3x + 222° = 540°$

$\Rightarrow 3x = 318°$

$\Rightarrow x = 106°.$

The Exterior Angles of a Polygon

The *exterior angles* of a polygon are formed by extending the sides in either direction.



The shaded angle is said to be an exterior angle of quadrilateral ABCD at vertices A, B, C and D.

The sum of the exterior angles of any polygon is always 360°.

Example 19. *A regular polygon has 15 sides. Calculate the size of each interior angle.*

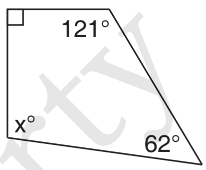
Solution. For a 15-sided polygon, each, exterior angle is 360° ÷ 15 = 24°

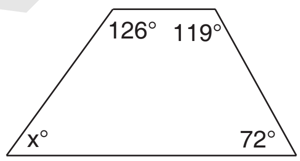
$\therefore$ Each interior angle is 180° – 24° = 156°.

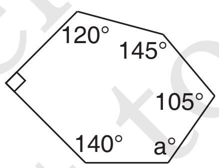
Note: A regular polygon has equal interior angles and equal sides.

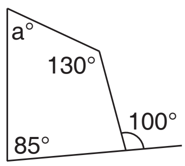
EXERCISE 5.7

1. Find the sum of the interior angles of:
   (*a*) a quadrilateral (*b*) a pentagon
   (*c*) a hexagon (*d*) an octagon
2. Find the value of the unknown in:

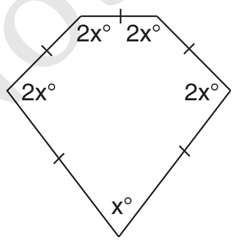
(*a*) 

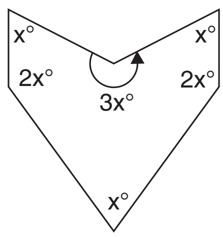
(*b*) 

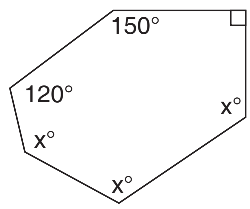
(*c*) 

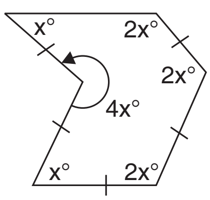
(*d*) 

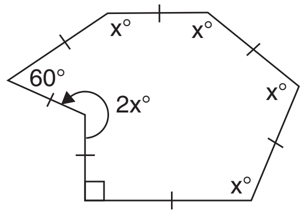
3. Find the value of *x* in each of the following, giving a reason.

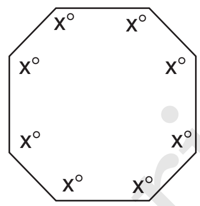
(*a*) 

(*b*) 

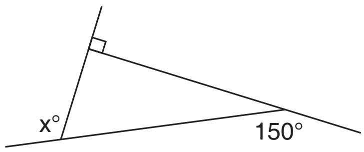
(*c*) 

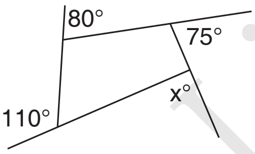
(*d*)


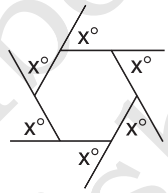
(*e*)


(*f*)


4. Solve for x:

(*a*)


(*b*)


(*c*)


5. Calculate the size of each interior angle of these regular polygons:

(*a*) with 5 sides (*b*) with 8 sides

6. Calculate the number of sides of a regular polygon given that an exterior angle is:

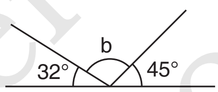
(*a*) 45° (*b*) 15° (*c*) $\frac{1}{2}^{\circ}$

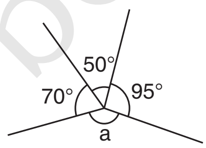
7. Calculate the number of sides of a regular polygon with an interior angle of:

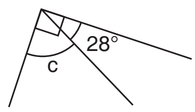
(*a*) 120° (*b*) 150° (*c*) 179°

REVIEW EXERCISE

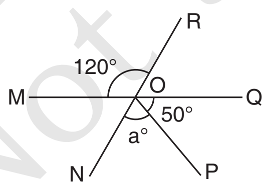
1. Calculate the size of each of the lettered angles in the diagrams below.

(*a*)


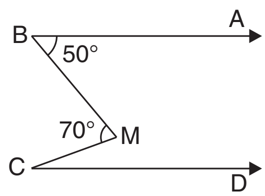
(*b*)


(*c*)


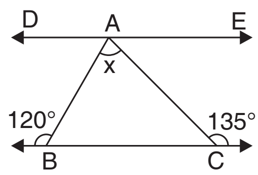
2. Find the value of a in the figure below.



Q. 2



Q. 3



Q. 4

3. In the above diagram, AB || CD, ∠ABM = 50°, ∠BMC = 70°. Find ∠MCD.

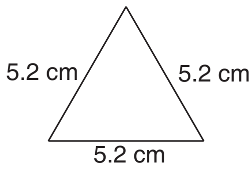
4. In the above diagram, DE || BC. Find the value of angle x.

5. Name the types of following triangles:

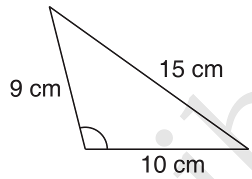
(*a*) ΔXYZ with $\angle Y = 90°$ and $XY = YZ$.

(*b*) ΔLMN with $\angle L = 30°$, $\angle M = 70°$ and $\angle N = 80°$.

6. Name each of the following triangles in two different ways: (you may judge the nature of the angle by observation)



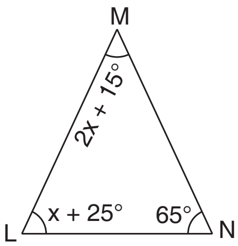
(a)



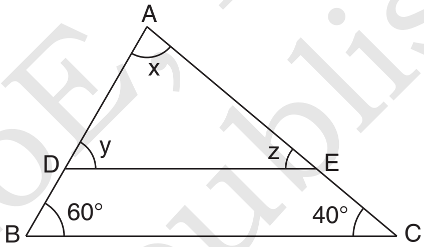
(b)

7. Find the value of x, y, z in the following triangles:

(*a*)



(*b*)



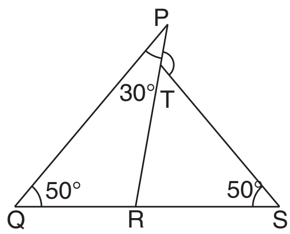
8. In the given diagram, find $\angle PTS$ and $\angle PRS$.

9. Show that each of the following is a perfect square. Also find the number whose perfect square is the given number.

(*a*) 324 (*b*) 4489

10. By Prime Factorisation method, find the square root of these numbers.

(*a*) 5184 (*b*) 7056



Q. 8

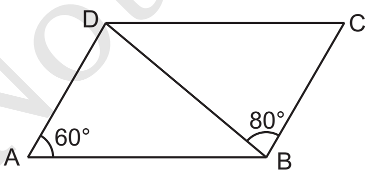
11. Which of the following are Pythagorean triples?

(*a*) {1, 2, 3} (*b*) {20, 48, 52} (*c*) {9, 12, 15} (*d*) {12, 16, 18}

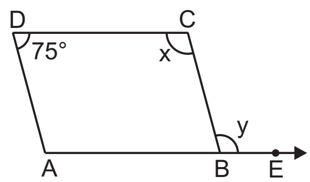
12. Find k if the following are Pythagorean triples:

(*a*) {15, 20, k} (*b*) {k, 45, 51} (*c*) {11, k, 61}

13. In parallelogram ABCD in the figure, $\angle DAB = 60°$ and $\angle DBC = 80°$. Find, $\angle ABD$.



Q. 13

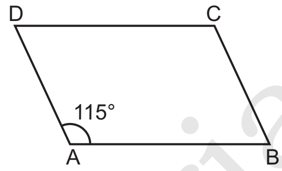


Q. 14

14. ABCD is a parallelogram in which $\angle ADC = 75°$ and side AB is produced to point E as shown in the figure. Find $(x + y)$.

15. Two opposite angles of a parallelogram are $(3x - 2)°$ and $(50 - x)°$. Find the measure of each angle of the parallelogram.

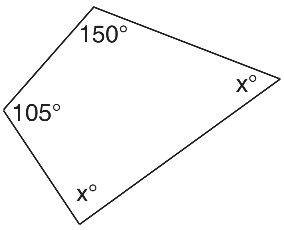
16. In a ||gm ABCD, if ∠A = 115°, find ∠B, ∠C and ∠D.



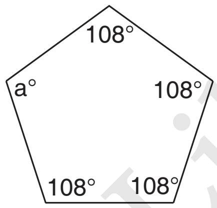
Q. 16

17. Find the value of the unknown in:

(*a*)



(*b*)



18. Calculate the size of each interior angle of these regular polygons:

(*a*) with 10 sides (*b*) with 20 sides

(*c*) with 100 sides (*d*) with n sides

19. Calculate the number of sides of a regular polygon given that an exterior angle is 2°.

20. Calculate the number of sides of a regular polygon with an interior angle of 175°.

MULTIPLE CHOICE QUESTIONS (MCQs)

1. At 3 o'clock, the angle between hour and minute hand is:

(*a*) 30° (*b*) 60° (*c*) 90° (*d*) 180°

2. An angle equal to its supplementary angle is:

(*a*) 45° (*b*) 90° (*c*) 180° (*d*) None of these

3. If angles $4x$ and $2x$ form a linear pair, then x is:

(*a*) 15° (*b*) 120° (*c*) 60° (*d*) 30°

4. If a transversal intersects two parallel lines, then:

(*a*) corresponding angles are equal

(*b*) alternate interior angles are equal

(*c*) co-interior angles are supplementary

(*d*) all of the above

5. Two adjacent angles:

(*a*) have a common vertex (*b*) have a common arm

(*c*) do not overlap (*d*) all of the above

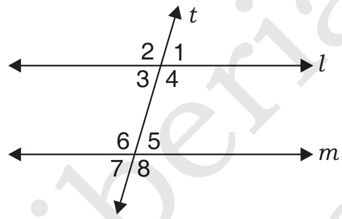
6. Angles $(5x - 10°)$ and $(2x - 20°)$ are co-interior angles when a transversal intersects two parallel lines. Their measures are:

(*a*) 150°, 30° (*b*) 140°, 40° (*c*) 100°, 80° (*d*) 120°, 60°

7. Two parallel lines are intersected by a transversal, so co-interior angles are $(3x + 20°)$ and x. Find the value of x.

(*a*) 60° (*b*) 120° (*c*) 140° (*d*) 40°

8. In the given diagram, lines l and m are non-parallel. Line t is a transversal cutting lines l and m at two distinct points. Which of the following holds true?



Q. 8

(*a*) $\angle 1 = \angle 3$ (*b*) $\angle 4 = \angle 6$

(*c*) $\angle 1 = \angle 7$ (*d*) None of these

9. Triangle having one angle 120° is called:

(*a*) Acute-angled Δ (*b*) Right-angled Δ

(*c*) Obtuse-angled Δ (*d*) None of these

10. If sides of the triangle are 4.2 cm, 3.5 cm and 4.2 cm, such Δ is called:

(*a*) Scalene (*b*) Equilateral

(*c*) Isosceles (*d*) Obtuse-angled

11. If the angles of a Δ are $(2x + 5°)$, $(3x + 10°)$, $(x + 15°)$, then x =

(*a*) 25° (*b*) 30° (*c*) 90° (*d*) 180°

12. If an exterior angle of a triangle is 120° and one of the opposite interior angles is 60°, then another opposite interior angle is equal to:

(*a*) 60° (*b*) 80° (*c*) 120° (*d*) 40°

RECAP AT A GLANCE

- Angles are formed when two lines meet.
- The sum of angles on a straight line is 180°.
- A line that intersects two or more lines (not necessarily parallel lines) at *distinct* points is called a *transversal.*
- An *acute-angled* triangle has *all its angles less than 90°.*
- A *right-angled* triangle has an angle of 90°.
- An *obtuse-angled* triangle has one angle greater than 90°.
- The sum of three interior angles of a triangle is always 180°.
- The exterior angles of a triangle always add up to 360°.
- The sum of consecutive interior and exterior angle is supplementary.
- The hypotenuse is always the longest side.
- The other two sides adjacent to the right angle are called base and perpendicular.
- Any closed figure/curve made up of only line segments is called a polygon.

❑ ❑ ❑